Hyperspectral unmixing with spectral variability using adaptive bundles and double sparsity – Complementary results

Tatsumi Uezato(1), Mathieu Fauvel(2) and Nicolas Dobigeon(1)

E-mail: {Tatsumi.Uezato, Nicolas.Dobigeon}@enseeiht.fr, mathieu.fauvel@ensat.fr

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(1) University of Toulouse, IRIT/INP-ENSEEIHT, 31071 Toulouse Cedex 7, France

(2) University of Toulouse, DYNAFOR/INP-ENSAT, 31326 Castanet Tolosan, France

Abstract

This supplementary material provides additional description of the proposed method and results introduced in [1]. Section I shows the minimization of each column in MEMM$_s$. Section II shows the parameter sensitivity of MEMM and MEMM$_s$ in simulated data. Section III shows SRE estimated by using the support extracted by the methods.

I. Minimization of each column in MEMM$_s$

The minimization of MEMM$_s$ is written as

$$\begin{aligned}&\min_{\mathbf{B}_i,\mathbf{a}_i} \frac{1}{2}\|\mathbf{E}\mathbf{B}_i\mathbf{a}_i - \mathbf{y}_i\|_2^2\\ &\text{s.t. } \forall k, \forall i,\ a_{ki} \geq 0, \quad \sum_{k=1}^{K} a_{ki} = 1, \quad \|\mathbf{a}_i\|_0 \leq v,\\ &\|\mathbf{b}_{ki}\|_0 \leq 1, \quad \mathbf{b}_{ki} \succeq 0.\end{aligned} \tag{1}$$

The minimization of $\mathbf{b}$ in MEMM$_s$ is different to MEMM in that it needs to select at most one coefficient value for each column of $\mathbf{b}$. In MEMM$_s$, each column is minimized separately. The objective function can be written as

$$\begin{aligned}&\min_{\mathbf{b}_{ki}} \frac{1}{2}\|(\sum_{j\neq k}^{K} \mathbf{E}_j\mathbf{b}_{ji}a_j + \mathbf{E}_k\mathbf{b}_{ki}a_{ki}) - \mathbf{y}_i\|_2^2\\ &\min_{\mathbf{b}_{ki}} \frac{1}{2}\|(\mathbf{E}_k \odot a_{ki})\mathbf{b}_{ki} - (\mathbf{y}_i - \sum_{j\neq k}^{K} \mathbf{E}_j\mathbf{b}_{ji}a_{ji})\|_2^2.\end{aligned} \tag{2}$$

Thus, the minimization of $\mathbf{b}_{ki}$ can be written as:

$$\begin{aligned} &\min_{\mathbf{b}_k} \frac{1}{2}\|(\mathbf{E}_k \odot a_{ki})\mathbf{b}_{ki} - \Psi\|_2^2 \\ &\text{s. t. } \forall k : \mathbf{b}_k \succeq 0, \quad \|\mathbf{b}_k\|_0 \leq 1 \end{aligned} \tag{3}$$

where $\Psi = \mathbf{y}_i - \sum_{j\neq k}^{K} \mathbf{E}_j \mathbf{b}_{ji} a_{ji}$. This problem can be minimized using the same approach with MEMM.

II. Parameter sensitivity of MEMM and MEMM$_s$

The sensitivity of parameters (λ_a and λ_m) in MEMM and MEMM$_s$ were shown in Fig. 1 and Fig. 2. MEMM and MEMM$_s$ showed the similar pattern in both SIM1 (40dB) and SIM2 (40dB). The following points were highlighted: 1) The large values (>5) of λ_a and λ_m led to the lower SRE. On the other hand, the large value of λ_a led to the smaller number of nonzero abundances. 2) The error in support could be minimized by selecting the optimal value (i.e. 1) of λ_a. 3) SRE was largest in MEMM when a certain level of sparsity ($\lambda_a = 0.1$) was imposed. These points showed that λ_a and λ_m may be selected by choosing the largest value in λ_a before SRE sharply degrades.

III. SRE estimated by using the support extracted by the methods

Abundances estimated by sparse unmixing may overly shrink because of the sparsity constraints. It is possible to extract the support of endmember spectra at first and then apply FCLS using the extracted support. In this experiment, the support of endmember spectra was firstly extracted by each method. The selected endmember spectra were then used with FCLS in order to estimate abundances. SRE calculated from the estimated abundances are shown in Table I and II. The results show that SREs derived from all methods were improved by using FCLS with the extracted support.

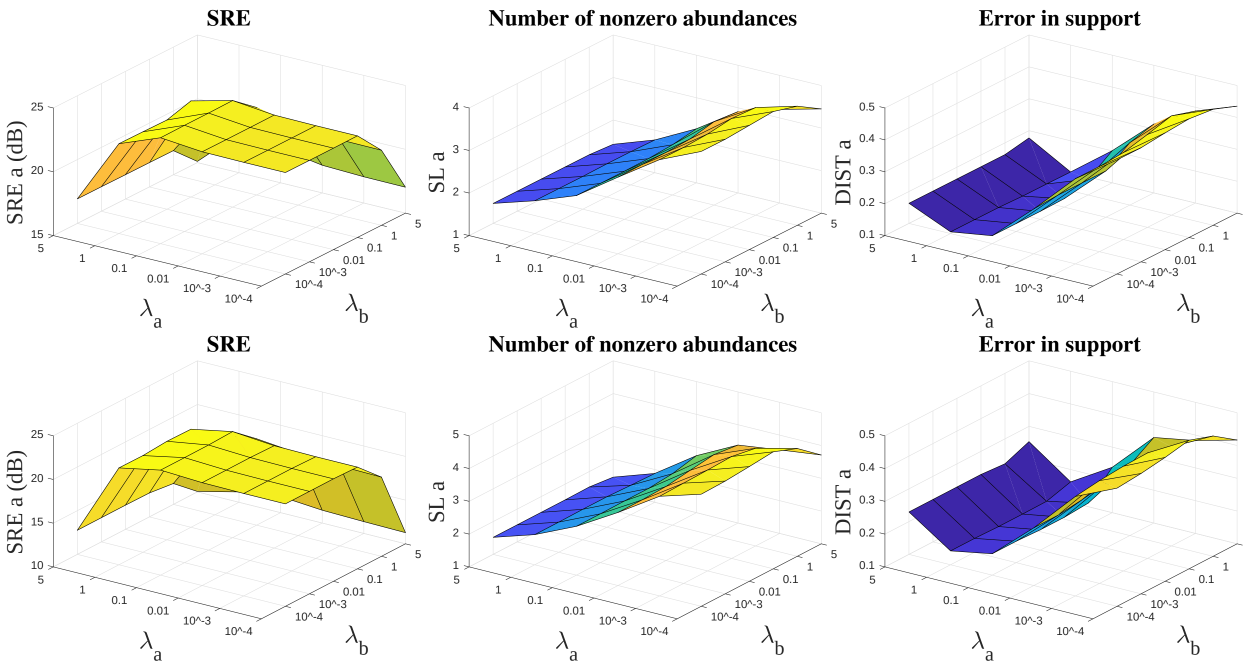

Fig. 1: Parameter sensitivity of MEMM in SIM1 and SIM2. First row shows results derived from SIM1. Second row shows results derived from SIM2.

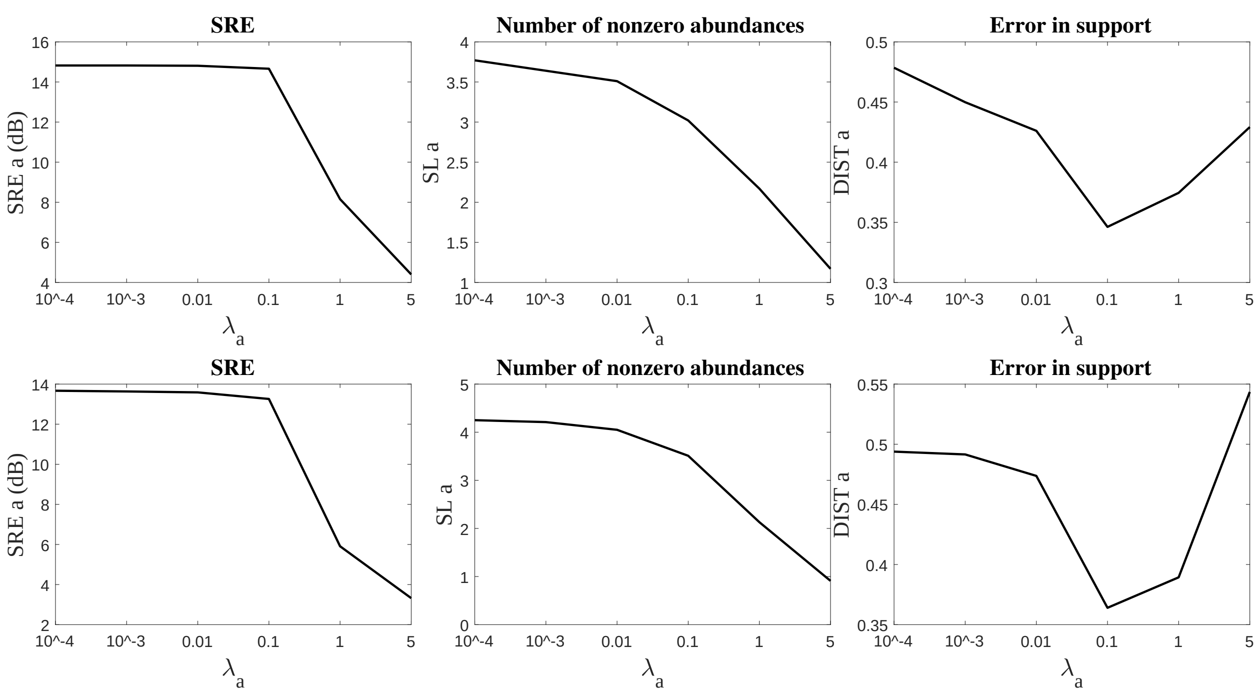

Fig. 2: Parameter sensitivity of MEMM$_s$ in SIM1 and SIM2. First row shows results derived from SIM1. Second row shows results derived from SIM2.

TABLE I: SRE per endmember class

	SNR	FCLS	AAM	SUNSAL	Group lasso	Elitist lasso	MEMM_s	MEMM
SIM1	30dB	19.1776	18.776	**19.7951**	19.5744	19.4452	14.8188	19.2048
	40dB	24.6072	**26.2779**	24.9072	24.6389	24.6926	17.5873	25.5761
	50dB	30.1878	**34.2536**	30.6512	30.3744	30.0024	16.7415	31.0991
SIM2	30dB	16.8877	16.0336	16.9101	**18.0346**	16.61534	12.5427	17.101
	40dB	22.1391	19.3149	22.1178	**22.7514**	21.9871	16.4467	22.6151
	50dB	26.7998	22.7456	26.741	**27.4978**	26.7204	14.8885	27.3563

TABLE II: SRE per endmember spectrum

	SNR	FCLS	AAM	SUNSAL	Group lasso	Elitist lasso	MEMM_s	MEMM
SIM1	30dB	1.5041	**2.2043**	1.3301	1.6291	1.6119	-0.4124	1.3563
	40dB	3.4652	**4.5951**	3.1526	3.2068	3.1425	0.7416	4.2799
	50dB	4.9884	**15.2514**	5.4139	4.9516	4.8917	1.8995	5.425
SIM2	30dB	0.9486	-0.1532	0.967	1.196	0.9885	-0.8207	**1.3389**
	40dB	2.6949	1.476	2.4487	2.7543	2.6519	0.1191	**2.8829**
	50dB	4.4627	3.3514	4.3511	4.3606	4.3428	1.1896	**4.584**

References

[1] T. Uezato, M. Fauvel, and N. Dobigeon, "Hyperspectral unmixing with spectral variability using adaptive bundles and double sparsity," submitted. [Online]. Available: https://arxiv.org/abs/